DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

Remark on solution of Problem 299, by J. M. ARNOLD, Crompton, R. I.

The following method shows *why* there is only *one* solution, a fact not clearly discernable in the published solution on page 203, Vol. XV.

Let x, y, $3y-x$ be the sides in increasing order of magnitude. Then the area must be $2y-2x$. This leads to an equation of the first degree in x, from which $x=\frac{1}{2}y+\dfrac{16y}{3y^2+16}$.

Substituting for y, the numbers 1, 2, 3, etc., we find $y=4$, $x=3$. All other integral values of y will give fractional values for x, as the second term soon becomes less than $\frac{1}{2}$, and continues to diminish as y increases.

Hence, 3, 4, 5 are the sides of the only triangle satisfying the problem.

302. Proposed by PROF. R. D. CARMICHAEL, Anniston, Ala.

Prove that the system of equations

$$xu+5yv=2,$$
$$xv+yu=1,$$

have no integral solution except one of the unknowns be zero.

I. Solution by E. B. ESCOTT, Ann Arbor, Mich.

Squaring and adding, and multiplying second equation by 5,

$$(xu-5yv)^2+5(xv+yu)^2=9.$$

But, since $(xu-5yv)^2+5(xv+yu)^2=(xu+5yv)^2+5(xv-yu)^2$, we must have the equation

$$X^2+5Y^2=9,$$

satisfied for two sets of values (X, Y).

The only solutions of this last equation are

$$X=\pm2,\ \pm3, \quad Y=\pm1,\ 0,$$

i. e.,

$$xu-5yv=2, \quad xv+yu=1,$$
$$xu+5yv=\pm3, \quad xv-yu=0.$$

Whence, $2xu=5$ or -1, $2xv=1$, evidently impossible.

Therefore, the only possible solutions are

$$xu-5yv=2, \quad xu+5yv=\pm 2.$$

Whence, $2xu=4$ or 0, $10yv=0$ or -4; *i. e.*, either x, u, y, or v equals zero.

Solved, similarly, by G. B. M. Zerr and V. M. Spunar.

II. Solution by O. C. CARMICHAEL, Oxford, Ala.

If the square of the first equation be added to five times the square of the second equation, we have

$$x^2u^2+5x^2v^2+25y^2v^2+5y^2u^2=9.$$

Therefore, there is no integral solution in x, y, u, v except when one of the unknowns is zero; for, if they were all positive integers, $25y^2v^2$ itself would be greater than 9.

303. Proposed by PROF. R. D. CARMICHAEL, Anniston, Ala.

Evaluate the determinant

$$\begin{vmatrix} D_1 & x_1x_2 & x_1x_3 & \dots & x_1x_n \\ x_1x_2 & D_2 & x_2x_3 & \dots & x_2x_n \\ x_1x_3 & x_2x_3 & D_3 & \dots & x_3x_n \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ x_1x_n & x_2x_n & x_3x_n & \dots & D_n \end{vmatrix}$$

Solution by J. W. CLAWSON, Ursinus College, Collegeville, Pa.

$$\Delta=x_1x_2x_3\dots x_n \begin{vmatrix} D_1/x_1 & x_2 & x_3 & \dots & x_n \\ x_1 & D_2/x_2 & x_3 & \dots & x_n \\ x_1 & x_2 & D_3/x_3 & \dots & x_n \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ x_1 & x_2 & x_3 & \dots & D_n/x_n \end{vmatrix}$$

$$=\prod_1^n x_1 \begin{vmatrix} (D_1-x_1^2)/x_1 & 0 & 0 & \dots & (x_n^2-D_n)/x_n \\ 0 & (D_2-x_2^2)/x_2 & 0 & \dots & (x_n^2-D_n)/x_n \\ 0 & 0 & (D_3-x_3^2)/x_3 & \dots & (x_n^2-D_n)/x_n \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ x_1 & x_2 & x_3 & \dots & D_n/x_n \end{vmatrix}$$

Subtracting the last row from each row,